The Design of a Class of Microwave Filters Using Lumped and Distributed Elements

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Abstract—**The design and performance of ladder networks containing both lumped and distributed elements are described. The necessary and sufficient conditions on the coefficients of the characteristic polynomial have been obtained and a suitable low-pass to band-pass transformation has been developed. The design procedure was then applied to the design of microwave filters and the measured performances show good agreement with the theoretical predictions.**

I. Introduction

GENERAL networks containing lumped and distributed elements have been dealt with by numerous authors [1]–[3]. However, most of their results deal with the theoretical aspects of the problem. The work described in this paper has concentrated on ladder networks and the main aim is to obtain a design procedure to realize practical circuits. The procedure has been applied to various examples with successful results.

The advantage of these circuits over circuits containing distributed elements only is that the response in the harmonic frequency bands can be greatly reduced and the advantage over circuits containing lumped elements only is that a greater rate of cutoff can be achieved for the same order because of the presence of finite transmission zeros.

II. Theoretical Analysis

A two-variable prototype ladder network is shown in Fig. 1. The impedances of all the series elements are proportional to the frequency variable $s = \sigma + j\omega$, and the admittances of all the shunt elements are proportional to another frequency variable $\lambda = \sum + j\Omega$. The variables s and λ are related by $\lambda = f(s)$ and, in general, $f(s)$ can take various forms. In a prototype lumped/distributed network all the series elements are inductors and all the shunt elements are open-circuited lengths of commensurate transmission lines. In this case $f(s) = \tanh T_n s$ where T_n is the delay on each line.

The input impedance of the network in Fig. 1(a) is given by

$$Z_{in}(s,\lambda) = \frac{c_0 + c_1 s + c_2 s\lambda + c_3 s^2\lambda + \cdots c_n s^{(n+1)/2}\lambda^{(n-1)/2}}{d_0 + d_1\lambda + d_2\lambda s + d_3\lambda^2 s + \cdots d_{n-1}\lambda^{(n-1)/2}s^{(n-1)/2}} \quad (1)$$

for n odd, and

$$Z_{in}(s,\lambda) = \frac{c_0 + c_1 s + c_2 s\lambda + c_3 s^2\lambda + \cdots c_n s^{n/2}\lambda^{n/2}}{d_0 + d_1\lambda + d_2\lambda s + d_3\lambda^2 s + \cdots d_{n-1}\lambda^{n/2}s^{n/2-1}} \quad (2)$$

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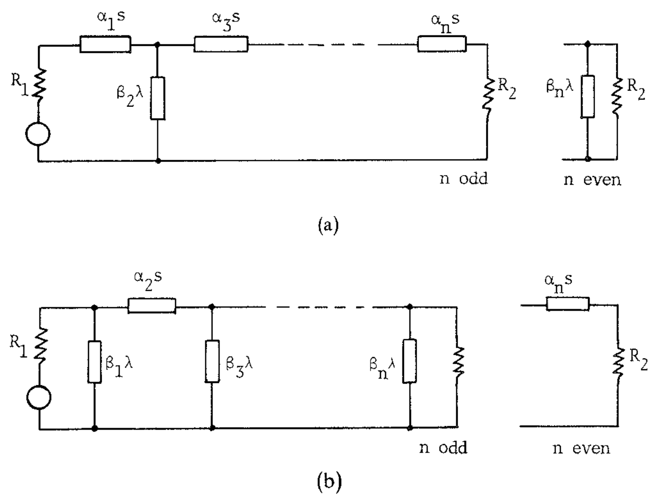

Fig. 1. Lumped/distributed ladder networks.

for n even. Similar forms can be obtained for the input impedance of the network in Fig. 1(b).

Without loss of generality, the constants c_0 and d_0 can be normalized to

$$c_0 = R_2 \quad \text{and} \quad d_0 = 1. \quad (3)$$

Conditions must exist on the constants c and d in order that the impedance function can be expanded in the continued fraction form

$$Z_{in}(s,\lambda) = \alpha_1 s + \frac{1}{\beta_2\lambda +} \frac{1}{\alpha_3 s + \cdots} \frac{1}{\alpha_n s +} \frac{1}{R_2} \quad (4)$$

with positive coefficients.

The necessary and sufficient conditions that the required continued fraction form exists are given by

$$\begin{aligned} c_1 d_1 &= c_0 d_2 + c_2 d_0 \\ c_1 d_3 + c_3 d_1 &= c_0 d_4 + c_2 d_2 + c_4 d_0 \\ &\;\;\vdots \\ c_n d_{n-2} &= c_{n-1} d_{n-1}. \end{aligned} \quad (5)$$

Conditions (5) together with (3) form $n+1$ conditions on the $2n+1$ coefficients c and d. Thus n coefficients could be chosen and the remaining $n+1$ coefficients are then determined. In other words, there are n conditions that can be imposed on the response of the network as in the case of a lumped ladder network.

The input scattering parameter $S_{11}(s,\lambda)$ can be obtained from

$$S_{11}(s,\lambda) = \frac{Z_{in} - R_1}{Z_{in} + R_1} \tag{6}$$

$$= \frac{a_0 + a'_1 s + a_1 \lambda + a_2 s\lambda + a'_3 s\lambda^2 + a_3 s^2\lambda + \cdots a_n s^{(n+1)/2}\lambda^{(n-1)/2}}{b_0 + b'_1 s + b_1 \lambda + b_2 s\lambda + b'_3 s\lambda^2 + b_3 s^2\lambda + \cdots b_n s^{(n+1)/2}\lambda^{(n-1)/2}} \tag{7}$$

for n odd, and a similar expression can be obtained for n even.

All the odd terms except the last are split into two terms and the total number of either the a or the b constants is $(3n + 1)/2$ for n odd and $(3n + 2)/2$ for n even.

The transfer scattering parameter $S_{21}(s,\lambda)$ is given by

$$S_{21}(s,\lambda) = \frac{2\sqrt{R_1 R_2}}{b_0 + b'_1 s + b_1 \lambda + b_2 s\lambda + b'_3 s\lambda^2 + b_3 s^2\lambda + \cdots b_n s^{(n+1)/2}\lambda^{(n-1)/2}}. \tag{8}$$

The relations between the b constants in (8) and the c and d constants in (1) are given by

$$b_i = c_i + d_i, \qquad \text{for } i \text{ even}$$

$$\left.\begin{aligned} b'_i &= d_i \\ b_i &= c_i \end{aligned}\right\}, \qquad \text{for } i \text{ odd.} \tag{9}$$

The b constants in the characteristic polynomial must satisfy the conditions in (9) and the c and d constants in turn must satisfy the conditions in (5).

III. The Network Response

When the relation between s and λ is known, the first step in the design procedure is to determine the b constants of (8) such that the desired response is obtained (n constraints) and at the same time conditions (3), (5), and (9) are satisfied [$(n + 1)/2$ constraints for n odd and $(n/2) + 1$ for n even].

As an example, an error minimization procedure was used to determine the b constants for a fifth-order equiripple response with cutoff at $\omega = 1$ rad/s, and a passband ripple of 1 dB. The normalized commensurate delay T_n was $\pi/4$ and in this case $\lambda = \tanh(\pi/4)s$. The 5 conditions on the passband are given by

a) The response at the first and second minima = 0 dB (2 conditions).
b) The response at the first and second maxima = 1 dB (2 conditions).
c) The response at cutoff ($\omega = 1$) = 1 dB (1 condition).

When the sum of the squares of the errors is less than 10^{-8}, the resulting b constants for equal normalized terminations are given by

$b_0 = 2$, $b'_1 = 8.09752$, $b_1 = 1.96973$,

$b_2 = 15.9499$, $b'_3 = 26.58673$, $b_3 = 3.30052$,

$b_4 = 15.49523$, $b_5 = 18.18617$.

The above constants relate to (8) when s and λ are interchanged.

When the continued fraction expansion is obtained, the circuit elements are given by

$C_1 = 2.359947$, $L_2 = 0.984665$, $C_3 = 3.402732$,

$L_4 = 0.985067$, $C_5 = 2.334839$.

The network is symmetrical and the small differences in the values of the symmetrical elements is due to the errors in the calculations. All the shunt elements C_1, C_3, and C_5 represent distributed capacitors and all the series elements L_2 and L_4 represent lumped inductors. The resulting response for the prototype network obtained is shown in Fig. 2.

When the b constants are known, a two-dimensional frequency plot can be drawn which will give the response for any relation between s and λ. The plot for the above example is shown in Fig. 3 and the response for any relation between s and λ can be found by plotting that relation and finding the intersection with the constant insertion loss contours. Examples of $\lambda = s$, $\lambda = \tanh s$, and $\lambda = \tanh(\pi/4)s$ are shown in Fig. 3.

IV. Frequency Transformation

After the low-pass prototype network is obtained, a frequency transformation step is required such that the relation

$$H_n(\omega_n,\Omega_n) = H(\omega,\Omega) \tag{10}$$

is satisfied, where the subscript n refers to the normalized quantities.

For the lumped/distributed case we also have

$$\Omega_n = \tan T_n\omega_n \quad \text{and} \quad \Omega = \tan T\omega. \tag{11}$$

To obtain the required transformation, the relations between the normalized and denormalized frequencies are

$$\omega_n = f_1(\omega) \quad \text{and} \quad \Omega_n = f_2(\Omega). \tag{12}$$

Furthermore, if the resulting network is to be realizable both $f_1(\omega)$ and $f_2(\Omega)$ must be positive-real functions.

From (11) and (12) we have

$$f_2(\Omega) = \tan\left[T_n(f_1(\omega))\right] = \tan\left[T_n f_1\left(\frac{1}{T}\tan^{-1}\Omega\right)\right]. \tag{13}$$

Thus in order that (10) is satisfied, the functions f_1 and f_2 are related by (13). It is not always possible to obtain positive-real functions f_1 and f_2 such that (13) is satisfied and (10) is satisfied for all values of ω.

A. Frequency Scaling

Scaling the frequency by a factor n_f can be easily achieved by the relations

$$\omega = n_f\omega_n$$

$$\lambda = \tan(n_f T_n\omega) = \tan T\omega. \tag{14}$$

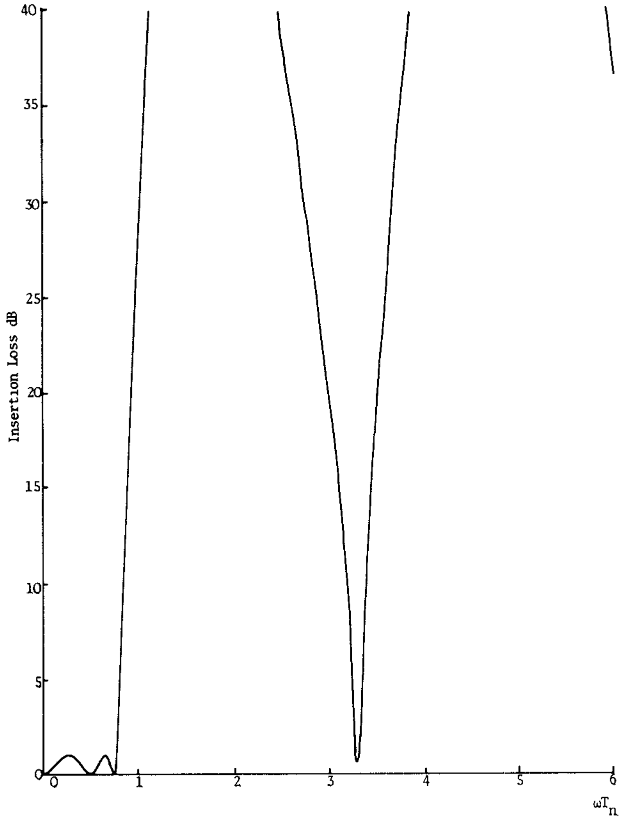

Fig. 2. Frequency response of fifth-order Chebyshev filter.

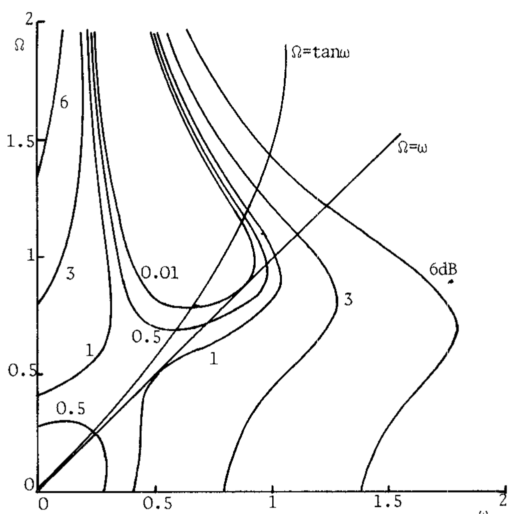

Fig. 3. Constant insertion-loss contours for third-order Chebyshev filter.

All the values of the series inductors are divided by n_f and the commensurate delay T of the scaled network is $n_f T_n$.

In this case, both (10) and (13) are satisfied and the transformation is valid for all values of ω.

B. Low-Pass to Bandpass Transformation

In this case it is not possible to find two positive-real functions f_1 and f_2 to satisfy the above conditions. The alternative is to satisfy (10) at a number of discrete frequencies. This could be achieved by choosing

$$f_1(\omega) = \frac{\omega_{nc}\omega_0}{\omega_2 - \omega_1}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$

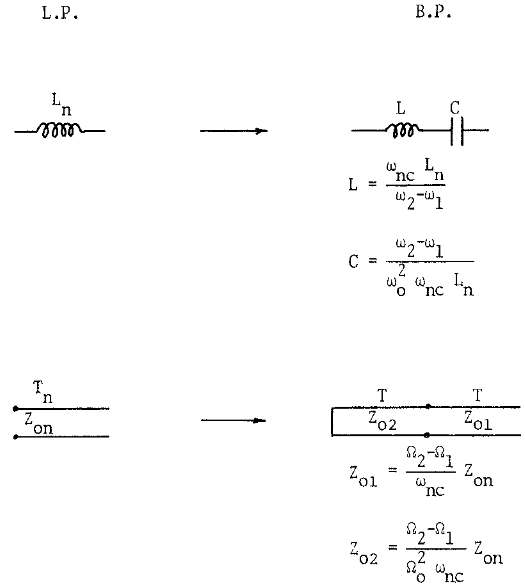

Fig. 4. Low-pass to bandpass transformations.

and

$$f_2(\Omega) = \frac{\Omega_{nc}\Omega_0}{\Omega_2 - \Omega_1}\left(\frac{\Omega}{\Omega_0} - \frac{\Omega_0}{\Omega}\right) \tag{15}$$

where ω_{nc} is the normalized cutoff frequency, $\omega_0^2 = \omega_1\omega_2$, ω_1 and ω_2 are the cutoff frequencies of the bandpass response, $\Omega_{nc} = \tan T_n\omega_n$, $\Omega_1 = \tan T\omega_1$, $\Omega_2 = \tan T\omega_2$, and $\Omega_0^2 = \Omega_1\Omega_2 = (\tan T\omega_0)^2$.

The condition $\Omega_0^2 = \Omega_1\Omega_2$ will determine the required value of T, the commensurate delay of the bandpass network. This is obtained by solving the equation

$$\tan T\omega_1 \tan T\omega_2 = \tan^2 T\omega_0. \tag{16}$$

There are several solutions to (16). Each solution will give a set of transformed impedance values and the final choice of T will be such that the most practical impedance values are obtained.

The resulting impedance transformations of the circuit elements are shown in Fig. 4.

The above transformation ensures that (10) is satisfied only at $(\omega_n = 0, \omega = \omega_0)$, $(\omega_n = \omega_{nc}, \omega = \omega_1)$, and $(\omega_n = \omega_{nc}, \omega = \omega_2)$.

The normalized cutoff frequency ω_{nc} can be chosen as either the 3-dB cutoff frequency, or the transmission zero frequency, or any other convenient value.

V. Practical Examples

Example I: A lumped/distributed third-order, maximally flat, ladder filter is designed to meet the following specifications:

$R_1 = R_2 = 50\ \Omega$ and the 3-dB cutoff frequencies are at 2.823 GHz and 3.183 GHz ($f_0 = 3$ GHz).

The b constants were determined for $T_n = 1$ and the elements of the low-pass prototype were then calculated.

Equation (16) was then solved for T and the value of T for the most practical values of the circuit elements was found to be 0.20485 ns.

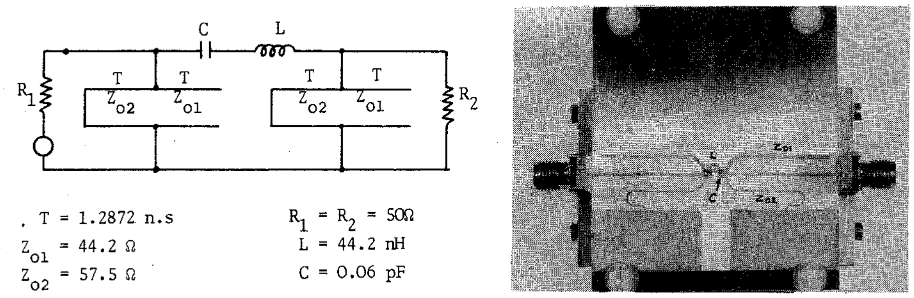

Fig. 5. Final bandpass circuit.

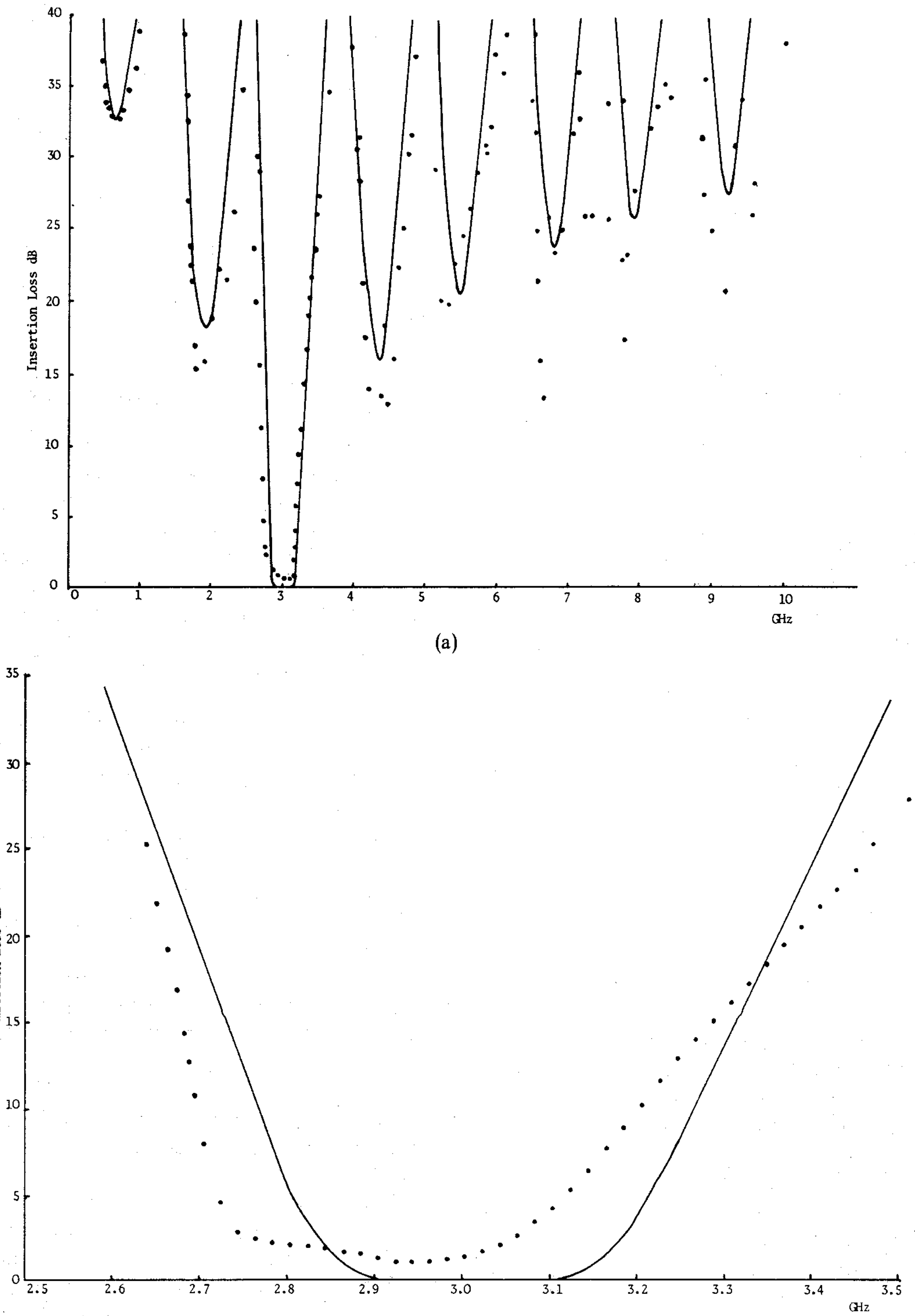

Fig. 6. (a) Theoretical response and measured points of the designed filter. (b) Expanded theoretical passband of designed filter and measured points.

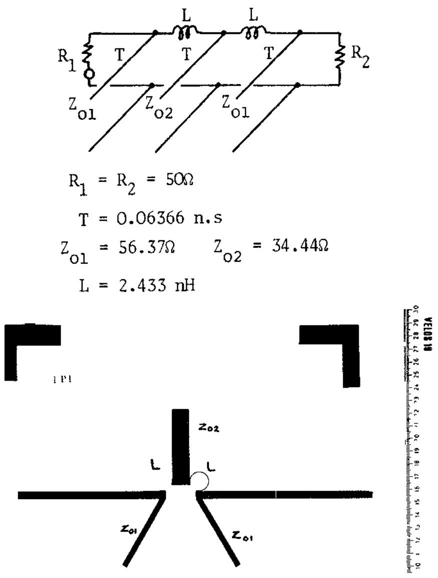

Fig. 7. Final circuit of fifth-order filter (Example II).

Fig. 5 shows the final bandpass circuit and a photograph of the actual filter constructed in microstrip form.

Fig. 6 shows the theoretical response of the filter and the measured points.

Example II: A fifth-order low-pass equiripple filter was designed with 0.5-dB passband ripple and a cutoff frequency of 3 GHz.

First the b constants were determined for the normalized prototype with $T_n = 1.2$ to satisfy the equiripple criteria. The circuit elements were then calculated for the required filter.

Fig. 7 shows the final circuit. The theoretical response and measured points are shown in Fig. 8.

VI. Conclusions

A successful design procedure has been developed for the design of lumped/distributed ladder networks. These networks have useful applications in the lower microwave range where the lumped elements can have reasonable performance. More accurate design procedure for lumped elements will be required if high-order filters of this type are to be designed.

As mentioned before, the advantage of a lumped/distributed filter over a purely distributed one is that attenuation can be obtained in the harmonic frequency bands. The advantage over a purely lumped filter is the higher rate of cutoff due to the finite transmission zeros.

The main disadvantage of the lumped/distributed circuit is the practical difficulties encountered in designing and producing the lumped elements at microwave frequencies. Various empirical formulae exist [4]–[6] for the design of these elements but none are accurate enough for the accurate prediction of their performance. Furthermore, lumped elements produce higher losses than distributed ones. The lumped elements are believed to be responsible for the insertion loss at the center frequency of the filter and for the slight shift in the cutoff frequencies. The measured attenuations in the harmonic bands are generally lower than the theoretical values; this is thought to be due to the coupling between the elements which exists in microstrip circuits.

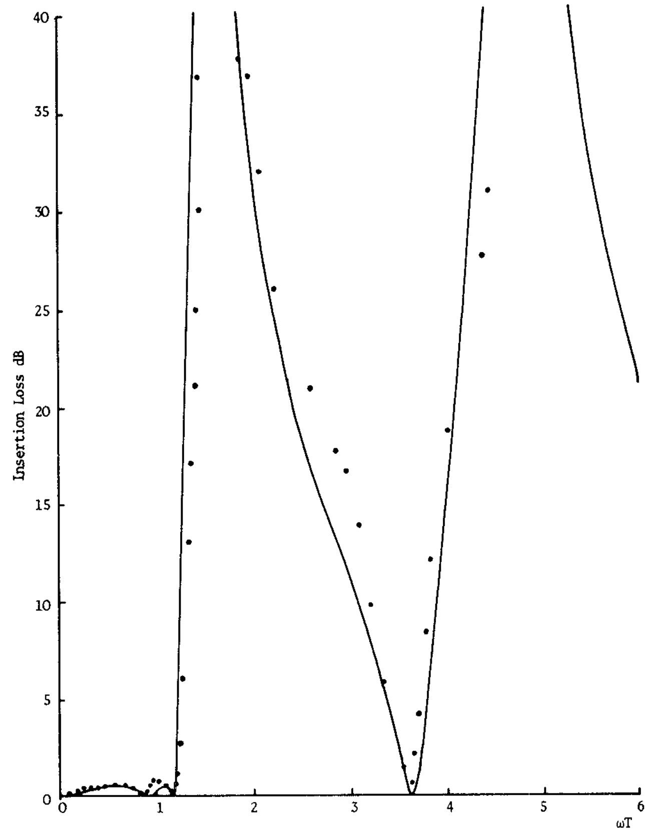

Fig. 8. Theoretical response and measured points of fifth-order Chebyshev filter (Example II).

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